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Steady flow in pipes of equilateral triangular cross-section in magnetic field

Dr. Anand Swrup Sharma
Associate Professor, Dept. of Applied Sciences, Ideal Institute of Technology, Ghaziabad, India
Email: Sharma.as09@gmail.com

ABSTRACT: In this paper we have investigate the Steady flow in pipes of equilateral triangular cross–section in magnetic field. We have investigated the velocity, volumetric flow and vortex lines.

KEY WORDS: Steady flow, Equilateral triangular cross section, incompressible fluid, pipes and magnetic field.

NOMENCLATURE

u =Velocity component along x – axis
v = Velocity component along y – axis
w (x , y) = Velocity in x-y plane
t = the time
ρ = The density of fluid
P = the fluid pressure
K= the thermal conductivity of the fluid
μ = Coefficient of viscosity
υ = Kinematic viscosity
Q = the volumetric flow
Ω_x = Vorticity component in x - direction
Ω_y = Vorticity component in y - direction
Ω_z = Vorticity component in z - direction

INTRODUCTION

We have investigated the Steady flow in pipes of equilateral triangular cross–section in magnetic field. Attempts have been made by several researchers i.e. S. I. Chernyshenko [1] an approximate method of determining the vorticity in the separation region as the viscosity tends to zero. P. Cherukat, J. B. McLaughlin & Graham A.L. [2] the inertial lift on a rigid sphere translating in a linear shear flow fluid. K. Chida [3] Heat transfer in steady laminar pipe flow with liquid solidification. S. Chikh, A. Boumedien, K. Bouhadef & G. Lauriat [4] Analytical solution of non-Darcian forced convection in an annular duct partially filled with a porous medium. S. Childress [5] Solutions of Euler's equations containing finite eddies. C. Chongsheng, A. R. Mohammad & S. T. Edriss [6] The Navier-Stokes Equations on the rotating 2-D sphere: Gevrey relularity and asymptotic degrees of freedom. G. Chukkapali & Q. F. Turan [7] Structural parameters and Prediction of adverse pressure gradient Turbulent Flows an Improved K.F. Model. E. Cumber batch & T. Y. Wu [8] Cavity flow past a slender hydrofoil. O. Dauchot, private communication. F. Daviaud, J. Hegseth & P. Berge [9] Subcritical transition to turbulence in plane Couette flow. U. S. De. [10] Importance of mountain waves in aviation and weather Hazzard associated with it. S. C. R. Dennis & G-Z. Chang [11] numerical solutions for steady flow past a circular cylinder at Reynolds numbers up to 100. S. C. R. Dennis, M. Ng & P. Nguyen [12] Numerical solution for the steady motion of a viscous fluid inside a circular boundary using integral conditions. Y. Ding & M. Kawahara [13] linear stability of incompressible fluid flow in a cavity using finite element method. R. K.Dubey & R. G.Sharma [14] A Note on the flow of Visco-elastic fluids through a rectilinear Pipe having its cross section as a parallelogram with pressure gradient as any function of time. In this paper we have

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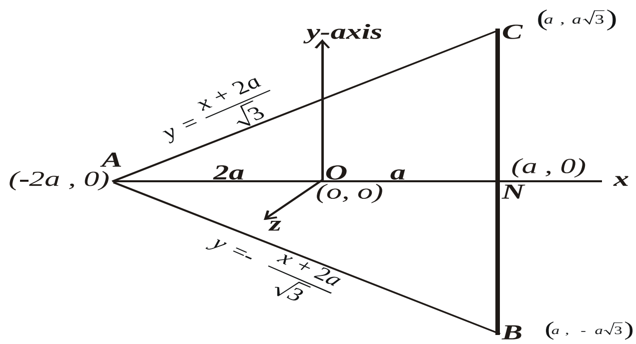
investigated the velocity, volumetric flow and vortex lines.

FORMULATION OF THE PROBLEM

Let z - axis be taken the direction of flow along the axis of the pipe. Then u = 0, v = 0 for steady and incompressible fluid the velocity component is independent of z . The equation of continuity.



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad ....(1)$$

$$But \quad u=0, \quad v=0 \quad \frac{\partial w}{\partial z}=0......(2)$$

$$\Rightarrow \quad w=w\ (x,y)........(3)$$

$$AB=BC=CA=2a\sqrt{3}, \quad AN=3a$$

Figure-1

i.e. w is independent of **z**

The Navier-Stokes equations of in the absence of body forces.

$$-\frac{\partial P}{\partial y}=0 \quad ....(4)$$

$$\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho}w=0 \Rightarrow \frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho\mu}\mu w=0............(5)$$

$$let \qquad \frac{\sigma B_0^2}{\rho\mu}=B^2$$

It is clear from (3) & (4) **P** is independent of **x** & y i.e. **p** is the Function of **z**

SOLUTION OF THE PROBLEM p = p (z) $\frac{\partial p}{\partial z}=\frac{dp}{dz}=$ Constant $=-$P

$$\mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-B^2 w\right]=\frac{dp}{dz} \quad \Rightarrow \quad \frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-B^2 w=-\frac{P}{\mu}.................(6)$$

$\left(D^2+D'^2-B^2\right)w=-\frac{P}{\mu}$ $\therefore$ $C.F. =\sum a_n\ e^{h_n x+h'_n y}$ Where h_n & h'_n are related by $h_n^{\,2}+h'^{2}_n-B^2=0$

and $P.I. = \frac{1}{D^2+D'^2-B^2}\left(-\frac{P}{\mu}\right) = \frac{P}{B^2\mu} \Rightarrow$ w (x, y) = $\sum_{n=1}^{\infty} a_n\ e^{h_n x+h'_n y}+\frac{1}{B^2\mu}P$ Where $h_n^2+h'^2_n=B^2$

Case –I using boundary conditions at y = $\frac{x+2a}{\sqrt{3}}$

$$-\frac{P}{B^2\mu}=\sum_{n=1}^{\infty} a_n\ e^{h_n x+h'_n\left(\frac{x+2a}{\sqrt{3}}\right)} \quad ....(7)$$

& at $y=-\frac{x+2a}{\sqrt{3}} \Rightarrow \sum_{n=1}^{\infty} a_n\, e^{h_n x - h_n'\left(\frac{x+2a}{\sqrt{3}}\right)} = -\frac{P}{B^2\mu}$ ....(8)

From (7) & (8) $\sum_{n=1}^{\infty} a_n \left[e^{h_n x + h'_n\left(\frac{x+2a}{\sqrt{3}}\right)} - e^{h_n x + h'_n\left(\frac{x+2a}{\sqrt{3}}\right)} \right] = 0 \Rightarrow e^{h_n x + h'_n\left(\frac{x+2a}{\sqrt{3}}\right)} = e^{h_n x - h'_n\left(\frac{x+2a}{\sqrt{3}}\right)}$

$$at\ x = a \quad e^{h_n a + \frac{h_n' a}{\sqrt{3}} + \frac{2a h_n'}{\sqrt{3}}} = e^{h_n a - \frac{a h_n'}{\sqrt{3}} - \frac{2a h_n'}{\sqrt{3}}} \Rightarrow h_n' = 0,\ h_n^2 + h'^2_n = B^2 \ \therefore h_n = B\ \&\ h_n' = 0$$

$$\sum_{n=1}^{\infty} a_n\, e^{Bx} + \frac{P}{B^2\mu} = 0 \Rightarrow \frac{P}{B^2\mu} = e^{Ba}\sum_{n=1}^{\infty} a_n \Rightarrow \sum_{n=1}^{\infty} a_n = -\frac{P}{B^2\mu} P\, e^{-Ba} \Rightarrow w_1(x,y) = \frac{P}{B^2\mu}\left[1 - e^{B(x-a)}\right]$$

Case - II $w\ (x,\ y) = 0\ \ at\ (-2a,\ 0)\ \ \&\ w(x,\ y) = 0\ at\ \ (a,\ a\sqrt{3})$

$$-\frac{P}{B^2\mu} = \sum_{n=1}^{\infty} a_n\, e^{-2\,a h_n} \quad ....(9)$$

$$-\frac{P}{B^2\mu} = \sum_{n=1}^{\infty} a_n\, e^{a h_n + a\sqrt{3}\ h'_n} \quad ....(10)$$

$$a h_n + a\sqrt{3}\ h'_n = -2a h_n \Rightarrow a\sqrt{3}\ h'_n = -3a h_n \Rightarrow h'_n = -\sqrt{3}\ h_n$$

$$h'^2_n + h_n^2 = B^2 \Rightarrow 4h_n^2 = B^2 \Rightarrow h_n = \pm\frac{B}{2} \quad \& \quad h'_n = \mp\frac{\sqrt{3}\,B}{2},\ \sum_{n=1}^{\infty} a_n = -\frac{P}{B^2\mu}\, e^{Ba}$$

$$w_2(x,y) = \frac{P}{B^2\mu}\left[1 - e^{\frac{B\left(x-\sqrt{3}\ y+2a\right)}{2}}\right]$$

Case – III $w\ (x,\ y) = 0\ \ at\ (-2a,\ 0)\ \ \&\ w(x,\ y) = 0\ at\ \ (a,\ -a\sqrt{3})$

$$-\frac{P}{B^2\mu} = \sum_{n=1}^{\infty} a_n\, e^{-2a h_n} \quad ....(11)$$

$$-\frac{P}{B^2\mu} = \sum_{n=1}^{\infty} a_n\, e^{a h_n - a\sqrt{3}\ h'_n} \quad ....(12)$$

On solving: $h_n = \frac{B}{2},\ \ h'_n = \frac{\sqrt{3}\,B}{2}\ \ \&\ \sum_{n=1}^{\infty} a_n = -\frac{P}{B^2\mu}\, e^{Ba} \Rightarrow w_3(x,y) = \frac{P}{B^2\mu}\left[1 - e^{\frac{B\left(x+\sqrt{3}\ y+2a\right)}{2}}\right]$

$$w(x,y) = \frac{P}{B^2\mu}\left[1 - \left\{ e^{\frac{B(x+2a)}{2}} . e^{-\frac{\sqrt{3}By}{2}} + e^{\frac{B(x+2a)}{2}}\ e^{\frac{\sqrt{3}By}{2}} \right\} - e^{B(x-a)}\right]$$

$$= \frac{P}{B^2\mu}\left[1 - e^{\frac{B(x+2a)}{2}} \left\{ e^{\frac{\sqrt{3}By}{2}} + e^{-\frac{\sqrt{3}By}{2}} \right\} - e^{B(x-a)}\right]$$

$$w(x,y)=\frac{P}{B^2\mu}\left[1-2\,Cosh\,\frac{\sqrt{3}\,B\,y}{2}\,e^{\frac{B(x+2a)}{2}}-e^{B(x-a)}\right]\text{ ....(13)}$$

The volumetric Flow

$$Q=\iint w\,(x,y)\,dx\,dy=\int_{x=-2a}^{a}\int_{y=-\left(\frac{x+2a}{\sqrt{3}}\right)}^{y=\left(\frac{x+2a}{\sqrt{3}}\right)}\frac{P}{B^2\mu}\left\{1-2\,Cosh\frac{\sqrt{3}\,By}{2}e^{\frac{B(x+2a)}{2}}-e^{B(x-a)}\right\}dx\,dy$$

$$=\frac{2P}{B^2\mu}\int_{-2a}^{a}\int_{0}^{\frac{x+2a}{\sqrt{3}}}\left\{1-2\,Cosh\frac{\sqrt{3}\,By}{2}\,e^{\frac{B(x+2a)}{2}}-e^{B(x-a)}\right\}dy\,dx$$

$$=\frac{2P}{B^2\mu}\int_{-2a}^{a}\left\{\frac{x+2a}{\sqrt{3}}-\frac{4}{\sqrt{3B}}\,Sinh\frac{B(x+2a)}{2}e^{\frac{B(x+2a)}{2}}-\left(\frac{x+2a}{\sqrt{3}}\right)e^{B(x-a)}\right\}dx$$

$$=\frac{1}{2}\int_{-2a}^{a}\left\{e^{B(x+2a)}-1\right\}dx=\frac{1}{2}\left\{\frac{e^{B(x+2a)}}{B}-x\right\}_{-2a}^{a}=\frac{1}{2}\left\{\frac{e^{3aB}}{B}-a-\frac{1}{B}-2a\right\}=\frac{1}{2}\left\{\frac{e^{3aB}}{B}-\frac{1}{B}-3a\right\}=\frac{1}{2B}\left\{e^{3aB}-1-3aB\right\}$$

$$Let\ \ I_1=\int_{-2a}^{a}e^{\frac{B(x+2a)}{2}}\,Sinh\frac{B(x+2a)}{2}\,.\,dx=\int_{-2a}^{a}e^{\frac{B(x+2a)}{2}}\left\{\frac{e^{\frac{B(x+2a)}{2}}-e^{-\frac{B(x+2a)}{2}}}{2}\right\}dx$$

$$Let\ \ I_2=\int_{-2a}^{a}\frac{(x+2a)}{\sqrt{3}}\,e^{B(x-a)}\,dx=\frac{1}{\sqrt{3}}\left[\left\{\frac{(x+2a)}{B}\,e^{B(x-a)}\right\}_{-2a}^{a}-\int_{-2a}^{a}\frac{1}{B}\,e^{B(x-a)}\,.\,dx\right]$$

$$=\frac{1}{\sqrt{3}}\left\{\frac{3a}{B}-\frac{1}{B^2}\left\{e^{B(x-a)}\right\}_{-2a}^{a}\right\}=\frac{1}{\sqrt{3}}\left[\frac{3a}{B}-\frac{1}{B^2}\left\{1-e^{-3aB}\right\}\right]=\frac{1}{\sqrt{3}}\left[\frac{3a}{B}-\frac{1}{B^2}+\frac{e^{-3aB}}{B^2}\right]=\frac{1}{\sqrt{3}\,B^2}\left[3aB-1+e^{-3aB}\right]$$

$$Let\ \ I_3=\int_{-2a}^{a}\frac{(x+2a)}{\sqrt{3}}\,dx=\left[\frac{(x+2a)}{2\sqrt{3}}\right]_{-2a}^{a}=\frac{(3a)^2}{2\sqrt{3}}=\frac{9a^2}{2\sqrt{3}}=\frac{3\sqrt{3}\,a^2}{2}$$

$$\therefore\ \ Q=\frac{2P}{\mu B^2}\left[I_3-\frac{4}{\sqrt{3B}}I_1-I_2\right]=\frac{2P}{\mu B^2}\left[\frac{3\sqrt{3}\,a^2}{2}-\frac{4}{\sqrt{3}\,B}.\frac{1}{2B}\left\{e^{3aB}-1-3aB\right\}-\frac{1}{\sqrt{3}\,B^2}\left\{3aB-1+e^{-3aB}\right\}\right]$$

$$Q=\frac{2P}{\mu B^2}\left\{\frac{9a^2}{2\sqrt{3}}-\frac{2}{\sqrt{3}\,B^2}\,e^{3aB}+\frac{2}{\sqrt{3}\,B^2}+\frac{6a}{\sqrt{3}\,B}-\frac{3a}{\sqrt{3}\,B}+\frac{1}{\sqrt{3}\,B^2}-\frac{1}{\sqrt{3}\,B^2}\,e^{-3aB}\right\}$$

$$=\frac{2P}{\mu B^2}\left\{\frac{9a^2}{2\sqrt{3}}+\frac{3}{\sqrt{3}\,B^2}+\frac{3a}{\sqrt{3}\,B}-\frac{2}{\sqrt{3}\,B^2}\,e^{3aB}-\frac{1}{\sqrt{3}\,B^2}\,e^{-3aB}\right\}$$

$$Q=\frac{2P}{\sqrt{3}\mu B^2}\left\{\frac{9a^2}{2}+\frac{3}{B}(a+\frac{1}{B})-\frac{1}{B^2}\left(2e^{3aB}-e^{-3aB}\right)\right\} \quad \text{.............} (14)$$

$$\text{Since} \quad w(x,y)=\frac{P}{\mu B^2}\left[1-2\,Cosh\frac{\sqrt{3}\,B\,y}{2}e^{\frac{B(x+2a)}{2}}-e^{B(x-a)}\right]$$

$$\text{Let} \quad \bar{q}=u\,\hat{i}+v\,\hat{j}+w\hat{k}=\frac{P}{\mu B^2}\left[1-2\,Cosh\frac{\sqrt{3}\,B\,y}{2}e^{\frac{B(x+2a)}{2}}-e^{B(x-a)}\right]\hat{k}$$

Let $\Omega_x,\ \Omega_y\ \&\ \Omega_z$ **are vorticity components**

$$\Omega_x=\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}=\frac{P}{\mu B^2}\left[-\sqrt{3}\,B\,Sinh\frac{\sqrt{3}\,B\,y}{2}e^{\frac{B(x+2a)}{2}}\right]=-\frac{\sqrt{3}\,P}{\mu B}\,e^{\frac{B(x+2a)}{2}}\,Sinh\frac{\sqrt{3}\,B\,y}{2}$$

$$\Omega_y=\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=-\frac{P}{\mu B^2}\left[-B\,Cosh\frac{\sqrt{3}\,B\,y}{2}e^{\frac{B(x+2a)}{2}}-Be^{B(x-a)}\right]=\frac{P}{\mu B}\left[Cosh\frac{\sqrt{3}\,B\,y}{2}e^{\frac{B(x+2a)}{2}}+e^{B(x-a)}\right] \quad \& \quad \Omega_z=0$$

Equation of vortex line $\frac{dx}{\Omega_x}=\frac{dy}{\Omega_y}=\frac{dz}{\Omega_z}$

$$\frac{dx}{-\frac{\sqrt{3}\,P}{\mu B}\,e^{\frac{B(x+2a)}{2}}\,Sinh\frac{\sqrt{3}\,B\,y}{2}}=\frac{dy}{\frac{P}{\mu B}\left[Cosh\frac{\sqrt{3}\,B\,y}{2}\,e^{\frac{B(x+2a)}{2}}+e^{B(x-a)}\right]}=\frac{dz}{0}$$

$$\text{Taking } I^{st} \text{ Two} \quad \frac{dx}{-\sqrt{3}\,e^{\frac{B(x+2a)}{2}}\,Sinh\frac{\sqrt{3}\,B\,y}{2}}=\frac{dy}{\left[Cosh\frac{\sqrt{3}\,By}{2}\,e^{\frac{B(x+2a)}{2}}+e^{B(x-a)}\right]}$$

$$\int \frac{\left(Cosh\frac{\sqrt{3}\,B\,y}{2}e^{\frac{B(x+2a)}{2}}+e^{B(x-a)}\right)}{e^{\frac{B(x+2a)}{2}}}dx+\sqrt{3}\int Sinh\frac{\sqrt{3}\,B\,y}{2}\,dy=C_1$$

$$\int Cosh\frac{\sqrt{3}\,B\,y}{2}\,dx+\int e^{\frac{B(2x-2a-x-2a)}{2}}\,dx+\sqrt{3}\ .\ \frac{2}{\sqrt{3}\,B}\,Cosh\frac{\sqrt{3}\,B\,y}{2}=C_1$$

$$\int Cosh\frac{B\left(x+2a\right)}{2}\,dx+\int e^{\frac{B(x-4a)}{2}}\,dx+\frac{2}{B}\,Cosh\frac{\sqrt{3}\,B\,y}{2}=C_1$$

$$\frac{2}{B}\,Sinh\frac{B\left(x+2a\right)}{2}+\frac{2}{B}\,e^{\frac{B(x-4a)}{2}}+\frac{2}{B}\,Cosh\frac{\sqrt{3}\,B\,y}{2}=C_1\Rightarrow Sinh\frac{B\left(x+2a\right)}{2}+e^{\frac{B(x-4a)}{2}}+Cosh\frac{\sqrt{3}\,B\,y}{2}=\frac{C_1 B}{2}=A$$

The first vortex line $e^{\frac{B(x-4a)}{2}} + Sinh\frac{\sqrt{3}\,B\,y}{2} + Cosh\frac{\sqrt{3}\,B\,y}{2} = A$ ....(15)

taking last two $dz = 0 \Rightarrow$ the second vortex line $z = B$ .... (16)

Table for Velocity: *Let* $P = 2, \quad \mu = .5,$ *are fixed and* $B = \sqrt{\frac{\sigma B_0^2}{\rho\mu}}, (x, y)$ *are change*

Table-1 (for velocity)

	(x,y)	$\left(-9,\frac{1}{6\sqrt{3}}\right)$	$\left(-12,\frac{1}{2\sqrt{3}}\right)$	$\left(-15,\frac{1}{\sqrt{3}}\right)$	$\left(-18,\frac{2}{\sqrt{3}}\right)$	$\left(-21,\frac{3}{\sqrt{3}}\right)$	$\left(-24,\frac{4}{\sqrt{3}}\right)$	$\left(-27,\frac{5}{\sqrt{3}}\right)$
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}} = 1$	$w(x,y)$	**2.209**	**3.589**	**3.8997**	**3.969**	**3.9896**	**3.996**	**3.999**
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}} = \frac{1}{2}$	$w(x,y)$	**.8315**	**8.795**	**12.519**	**14.203**	**15.025**	**15.45**	**15.683**
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}} = \frac{1}{3}$	$w(x,y)$	**-8.346**	**9.178**	**19.622**	**25.677**	**29.324**	**31.58**	**33.024**
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}} = \frac{1}{4}$	$w(x,y)$	**-25.164**	**1.914**	**21.41**	**34.21**	**42.82**	**48.71**	**52.82**
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}} = \frac{1}{6}$	$w(x,y)$	**-99.8**	**-42.65**	**.316**	**32.23**	**56.26**	**74.54**	**88.58**

CONCLUSION AND DISCUSSION

In this paper, we have investigated the velocity $w(x, y)$ by the table-1 of equation (13) between velocity and point (x, y), it is clear that the velocity $w(x, y)$ of increases in the interval $\left(-9, \frac{1}{6\sqrt{3}}\right) \le (x, y) \le \left(-27, \frac{5}{\sqrt{3}}\right)$ at the different values of $\sqrt{\frac{\sigma B_0^2}{\rho\mu}}$. Again the velocity $w(x, y)$ increases correspondingly in the interval $\left(-9, \frac{1}{6\sqrt{3}}\right) \le (x, y) \le \left(-27, \frac{5}{\sqrt{3}}\right)$ when $\sqrt{\frac{\sigma B_0^2}{\rho\mu}}$ decreases from1 to $\frac{1}{6}$. Negative sign of velocity $w(x, y)$ shows that the direction of flow is in opposite to the direction of motion of fluid. Also we have investigated the volumetric flow, vortex lines respectively by the equations (14), (15) & (16).

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